M1 Internship: Backward Responsibility in Counterexamples of Model Checkers

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Outline

1. The problem
2. Semivalues
3. Optimistic and pessimistic responsibilities
4. Characterisation of the optimistic responsibility
5. Complexity results

What is model checking? (at least in our case)

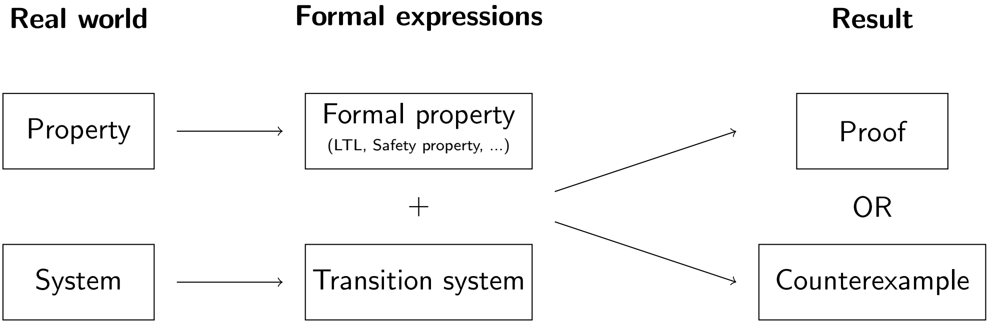


Figure: Model checking in a diagram

The problem: Intuition

"Backward Responsibility in Counterexamples of Model Checkers"

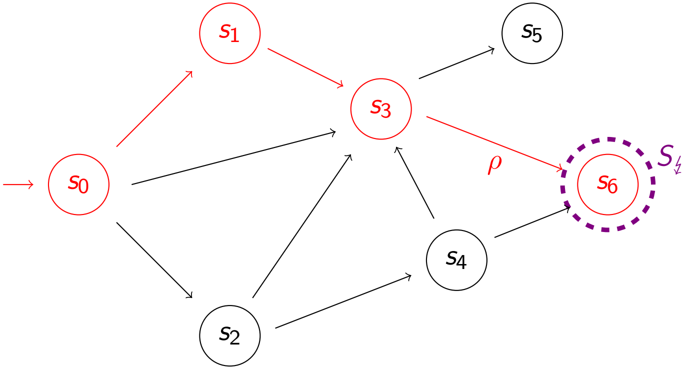


Figure: A transition system with a counterexample in red

The problem: Transition systems

Definitions:

- **Transition systems:** $TS = (S, \rightarrow, s_0)$, deterministic
- **Runs:** infinite sequence of states $\rho = \rho_0\rho_1 \ldots \in S^\omega$ where $\rho_0 = s_0$ and $\forall i \in \mathbb{N}\ \rho_i \rightarrow \rho_{i+1}$
- **Set of bad states:** $S_\lightning \subseteq S$
- **Counterexamples:** $\rho = \rho_0 \ldots \rho_k \in S^*$ such that it is the prefix of a run, $\rho_k \in S_\lightning$, $\rho_i \notin S_\lightning$ for $i \in \{0, \ldots, k-1\}$ and $\rho_i \neq \rho_j$ for all $i \neq j$ *ie.* they are loop-free.

Semivalues

- Finite set of *players* X
- *Coalitions*: $C \subseteq X$
- *Cooperative games*: $v\colon 2^X \to \mathbb{R}$
- Set of cooperative games on X: G^X

Definition (Semivalue)

Let X be a finite set of players with $n := |X|$. Then $\mathcal{R}\colon G^X \to X \to \mathbb{R}$ is a *semivalue* if there exists a weight vector $p = (p_0, \dots, p_{n-1})$ such that, for any game $v \in G^X$ and player $i \in X$, we have

$$\mathcal{R}(v, i) = \sum_{C \subset X \setminus \{i\}} p_{|C|}[v(C \cup \{i\}) - v(C)]$$

Semivalues

Definition

We call $p = (p_0, \ldots, p_{n-1})$ a *weight vector* if

$$\sum_{k=0}^{n-1} \binom{n-1}{k} p_k = 1.$$

Classical semivalues:

- The *Shapley value*: $p_k^{\mathcal{S}} := \frac{(n-k-1)!k!}{n!}$
- The *Banzhaf value*: $p_k^{\mathcal{B}} := \frac{1}{2^{n-1}}$

Intuition

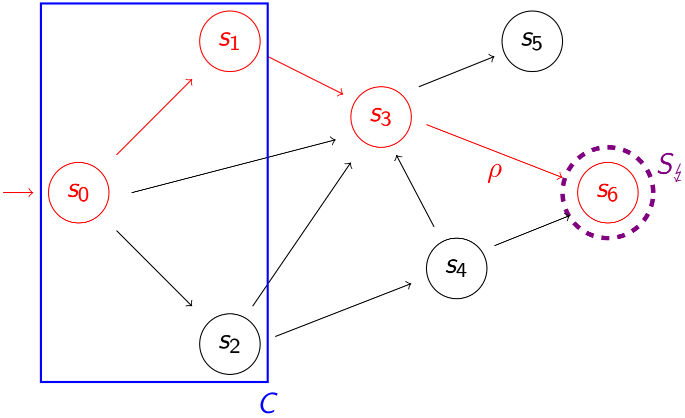
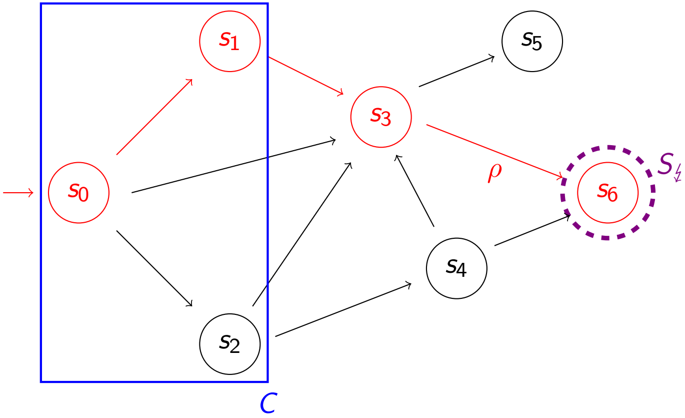


Figure: A transition system with a counterexample in red

Intuition



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➔ How to quantify the actions of $S \setminus (C \cup \rho)$?

Intuition

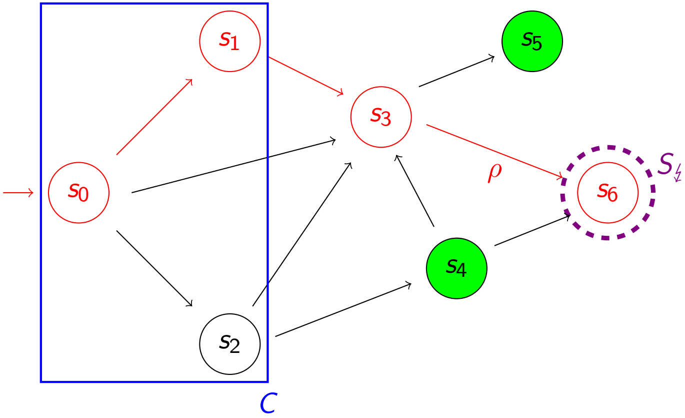


Figure: A transition system with a counterexample in red

➔ How to quantify the actions of $S \setminus (C \cup \rho)$?

Optimistic and pessimistic responsibilities

Safety games: $(S_{Safe}, S_{Reach}, \to, s_0, S_{\lightning})$

- Transition system: $(S, \to, s_0)$
- $S := S_{Safe} \uplus S_{Reach}$ and $S_{\lightning} \subseteq S$
- Winning condition of the form $\Omega_{S_{\lightning}} = \{\rho \mid \forall i \in \mathbb{N}\colon \rho_i \notin S_{\lightning}\}$
- A strategy for *Safe* is a function $\sigma\colon S_{Safe} \to S$ with $s \to \sigma(s)$ for all $s \in S_{Safe}$ (same for *Reach*)
- A strategy for *Safe* is winning if, for all strategies of *Reach*, the induced play is winning for *Safe*, *ie.* $\rho \in \Omega_{S_{\lightning}}$.

Optimistic and pessimistic responsibilities

$\mathcal{G}^{TS}_{\rho,S_{\lightning}}(C)$: safety game defined as $(C, S \setminus C, \rightarrow', s_0, S_{\lightning})$ where

- $\rightarrow'$ is $\rightarrow$ in which actions from ρ are "engraved" for $\rho \setminus C$.
- *Safe* controls C
- *Reach* controls $S \setminus C$

Definition (Optimistic and pessimistic cooperative games)

Let $C \subseteq S$.

Optimistic cooperative game: $$v_\top(C) = \begin{cases} 1 & \text{if player } \textit{Safe} \text{ wins } \mathcal{G}^{TS}_{\rho,S_{\lightning}}(C \cup (S \setminus \rho)) \\ 0 & \text{otherwise} \end{cases}$$

Pessimistic cooperative game: $$v_\perp(C) = \begin{cases} 1 & \text{if player } \textit{Safe} \text{ wins } \mathcal{G}^{TS}_{\rho,S_{\lightning}}(C) \\ 0 & \text{otherwise.} \end{cases}$$

Optimistic and pessimistic responsibilities

Now we can apply semivalues :-)

Remember how semivalues look like:

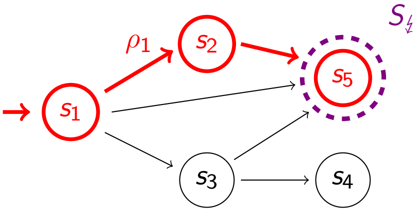
$$\mathcal{R}(v, i) = \sum_{C \subset X \setminus \{i\}} p_{|C|}[v(C \cup \{i\}) - v(C)]$$

Definition (Responsibility)

Let ρ be a counterexample, let $\mathcal{R}$ be a semivalue on G^S.

1. The *optimistic responsibility of s with respect to* $\mathcal{R}$ is $\mathcal{R}(v_\top, s)$.
2. The *pessimistic responsibility of s with respect to* $\mathcal{R}$ is $\mathcal{R}(v_\bot, s)$.

Example



s	$\mathcal{S}(v_\top, s)$	$\mathcal{S}(v_\bot, s)$	$\mathcal{B}(v_\top, s)$	$\mathcal{B}(v_\bot, s)$
s_1	1	0.5	1	0.5
s_2	0	0	0	0
s_3	0	0.5	0	0.5
s_4	0	0	0	0
s_5	0	0	0	0

Figure: Working example 3, run 1

Characterisation of the optimistic responsibility

Set of winning states: $\mathtt{WS}_\top := \{s \in S \mid v_\top(\{s\}) = 1\}$

Set of responsible states: $\mathtt{RS}_\top(\mathcal{R}) := \{s \in S \mid \mathcal{R}(v_\top, s) > 0\}$

Proposition

Let $\mathcal{R}$ be a semivalue with $\mathtt{Weights}_0(\mathcal{R}) > 0$. Then we have

$$\mathcal{R}(v_\top, s) > 0 \iff v_\top(\{s\}) = 1, \text{i.e. } \mathtt{RS}_\top(\mathcal{R}) = \mathtt{WS}_\top.$$

Characterisation of the optimistic responsibility

Theorem (Characterisation)

Let $\mathcal{R}$ be a semivalue, then there exists $K \in \mathbb{R}$ such that

- $\forall s \notin \mathtt{WS}_\top, \mathcal{R}(v_\top, s) = 0$
- $\forall s \in \mathtt{WS}_\top, \mathcal{R}(v_\top, s) = K$

and $K = \displaystyle\sum_{k=0}^{n-w} \binom{n-w}{k} p_k$ where $w := |\mathtt{WS}_\top|$.

Additionally, we have $\mathtt{WS}_\top \subseteq \rho$.

Complexity results

Optimistic case:

- Positivity problem: Linear time
- Threshold problem and computation problem: Quadratic time

Pessimistic case:

- Positivity problem: in NP (actually NP-complete)
- Threshold problem: in PSPACE
- Computation problem: in #P

Conclusion

Summary:

- Two notions of responsibility
- Both intuitive and effective (automatic repair)
- Simple characterisation for the optimistic responsibility
- Linear complexity for the optimistic responsibility
- Pessimistic responsibility is more complex

Other contributions:

- Quick implementation (coalition trees, attractor algorithm)
- Article submitted at AAAI
- Recursive responsibility, an inspiring fail
- Generalisation to LTL properties
- A conjecture tested for $n \leq 5$: Banzhaf and Shapley values give equivalent results
- And next...

References

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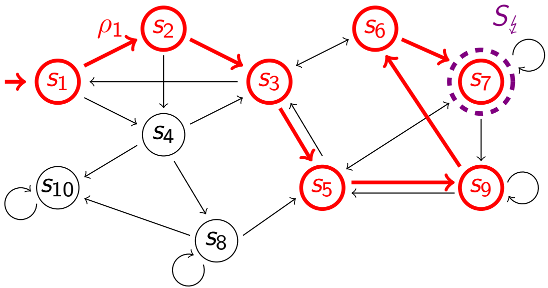
Thank you for your attention.

Example 2



s	$\mathcal{S}(v_\top, s)$	$\mathcal{S}(v_\bot, s)$	$\mathcal{B}(v_\top, s)$	$\mathcal{B}(v_\bot, s)$
s_1	0.1667	0.0238	0.1667	0.04
s_2	0.1667	0.0238	0.1667	0.04
s_3	0.1667	0.2238	0.1667	0.2
s_4	0	0.0571	0	0.12
s_5	0.1667	0.2238	0.1667	0.2
s_6	0.1667	0.2238	0.1667	0.2
s_7	0	0	0	0
s_8	0	0	0	0
s_9	0.1667	0.2238	0.1667	0.2
s_{10}	0	0	0	0

Figure: Working example 10, run 1